NORMAL TEMPERATURES (DAILY):

ARE IRREGULARITIES IN THE ANNUAL MARCH OF TEMPERATURE PERSISTENT?

By CHARLES F. MARVIN, Chief of Weather Bureau.

[Dated: Washington, Sept. 10, 1919.]

SYNOPSIS.—The data examined in this study consisted of daily values of mean temperature from several old records for stations in the northeastern portions of the United States, covering periods from 1778 to 1865. These were supplemented by 45-year records from widely scattered Weather Bureau stations covering the period 1871 to 1916. The observations were reduced to 52 weekly mean values, which were subjected to the harmonic analysis, resulting in the following conclusions:

1. The annual march of daily mean temperatures is a smooth curve devoid of secondary maxima and minima or of perceptible points of inflection.

2. Marked irregularities, such as connoted by terms like the January Thaw, the May Freeze, etc., do not persist nor have a real existence. When such irregularities still remain in the means of a long record they are simply residual imprints of a single or a few fortuitously recurrent extreme or unusual events at or near the time in question.

3. (*a*) The Fourier series is a simple but powerful and quite adequate mathematical equation for the analysis of the annual march of temperature and the determination of daily and other approximate normals, even from relatively short records. (*b*) From the examination thus far made of United States data it appears the series must stop at the second term, which seems very small or inconsequential for the northeastern section. (*c*) It is fully recognized that the ultimate normal curve of annual temperature may not be quite represented by the harmonic equation of the second order. It is believed, however, that any systematic departures which may ultimately be disclosed by very long records will not be of a periodic character. Accordingly, an equation of the following form seems likely to suffice for the most refined studies:

$$T_r = A_0 + a_1 \cos(\theta_r - \phi_1) + a_2 \cos(\theta_r - \phi_2) + f(r)$$

As far as known at present, $f(r) = 0$. The longest records now available here are insufficient to evaluate it.

4. Monthly means are quite inadequate for the analysis of the annual march of temperature because many important characteristics are lost in monthly summations. Moreover, uncertain corrections are required to equalize monthly intervals and allow for the "curvature" in the temperature changes within the month. The week as a subunit is a happy medium between the month and the pentad and is decidedly superior to both.

Meteorological literature contains a large number of discussions of the daily and seasonal march of temperature through the annual cycle. Familiar terms, such as the May Freeze, or Ice Saints, the January Thaw, Indian Summer, and the like, testify to the wide prevalence of the conviction in many minds that peculiar irregularities in the annual march of temperature persist or recur with greater or less certainty at particular times and seasons.

This paper is followed by a very interesting abstract of much of the literature on this question, prepared by the Weather Bureau librarian, Prof. C. F. Talman.

From time to time in the past the Weather Bureau, as has been the case with other meteorological services and independent observatories, has published so-called normal values of temperature and other data deduced from a careful discussion of the observations available at the time. As the length of the record has increased with the lapse of years the values of the "normals" have been "revised," a procedure or necessity which, in itself, discredits or makes incongruous the logical significance[1] of the term "normal."

The purpose of the present paper is to examine this whole question comprehensively. This is the more necessary because an examination of Prof. Talman's compendium clearly indicates that affirmations as to the reality of the Ice Saints and like peculiarities are based on a study of but a fragment of the data comprising the annual march of temperature.

PHYSICAL CONSIDERATIONS.

Throughout the entire present considerations all the phenomena of the *diurnal* march of temperature are wholly disregarded. The data and conclusions apply to only a daily mean temperature or other representative value, as a maximum or minimum temperature, or a value at some particular hour of a single day.

The temperature at any station on the earth's surface is dominated primarily by the effective local intensity of solar radiation. Confining attention to the major variations in effective solar radiation, which depend on the revolution of the earth around the sun and the inclination of its axis to the ecliptic, we see that this dominating control of daily average temperatures is purely a periodic sequence with all the characteristics of a harmonic function. We must, therefore, expect that the normal annual march of temperature can be represented more or less closely by a harmonic function—a Fourier series.

Modifying variations of the average daily temperature are clearly caused by progressive or seasonal variations in the phenomena of the general circulation of the air and other minor controls on local temperature. However, perhaps all of these minor controls are themselves a function of the dominating control by effective solar radiation, and therefore become periodic and harmonic.

In the long run, therefore, we should expect the normal annual march of temperature to be a smooth, regular curve, devoid of secondary maxima and minima values.

If, however, claims are set up that secondary maxim and minima exist and persist indefinitely at particular places on the annual curve, then a physical cause and explanation of such irregularities is needed. It is recognized that some explanations, based on systematic shifting of winds, have been offered in a single or a few cases, but this still leaves unexplained their physical cause, as likewise that of a number of other equally conspicuous irregularities which appear in the march of temperature when the relatively short records now available are examined in the customary way.

THE DATA IN GENERAL.

It is well known that the simple average values of the daily temperatures, even for a long, homogeneous series of observations, show many irregular sequences of secondary maxima and minima. With short records these irregularities are striking; with longer records the alternations are less developed and seem to tend to disappear as the number of observations increases, which tendency confirms expectations reached from physical considerations that ultimately the average of centuries of observa-

[1] See Monthly Weather Review, 1918, 46: 515.

tions of daily temperatures forms a smooth annual curve devoid of secondary maxima and minima.

Few full and authentic records anywhere in the world exceed a length of 150 years. In the United States, few records, even supplemented by those of the Weather Bureau, attain to a length of a hundred years. Many good records of 45 or more years are now available for Weather Bureau stations. A preliminary and tentative examination has been made of the frequency distribution and standard deviation of some of these records of temperature for winter months for the stations at Washington and Baltimore. The results indicate that the mean temperature for any one day is just as likely as not to differ from the normal by more than 6.°1 as less. The same data indicate that the probable error of the mean of 40 years' daily observations is more than ±1.°0; also the probable error of a weekly mean is about ±0.°4, and of a weekly mean of 15 years is about ±0.°7. These numerical data deduced from the observations strenghten confidence in the expectations reached from physical considerations and the cursory examination of the data and tend to show that the irregularities found in the daily means of present short records are largely, if not wholly, *accidental*, and therefore not persistent, and are chiefly caused by paucity of data. If this is true the features will be absent from the means of very long records.

Smoothing methods.—Without exception probably every one concedes that some method of eliminating the irregularities of such data as the direct average values in short records of daily temperatures is necessary in securing therefrom approximate normal values. The popular methods of plotting the observations and drawing smooth free-hand curves, as also the use of such smoothing formulae as $\frac{1}{4}$ (a + 2b + c) and others of a higher order, must be regarded as very unsatisfactory in comparison with the use of a mathematical equation embracing the whole cycle whenever the latter is available. Since for the purposes of the present problem the physical considerations mentioned, as also the experience of others, clearly indicated that the normal annual march of temperature is well represented by a harmonic equation of low order, this was adopted as the simplest and most dependable means available for the analysis and comparison of many temperature records to determine the reality of the irregularities claimed to exist by some. Expectations have been very fully verified and confirmed by the experience gained in the execution of the work itself and firmly establish the conviction that the harmonic equation of the first or second order is decidedly the best approximation available of the annual march of temperature. How this may be improved is indicated by the form of equation suggested in the synopsis.

To inspire, if need be, a fuller measure of confidence in the superiority of the mathematical curve over one drawn by hand, or over a result secured by some smoothing formula, it may be pointed out that the hand has no choice but to guide its way *crudely* among closely adjacent points located by more or less discordant data and without power of discernment of the control and influence which the data fixing more distant points should exercise. The same thing is true to a much greater degree in simple smoothing formulae commonly employed. Indeed, Ryd[2] has pointed out that an arbitrary smoothing formula becomes sufficient and adequate only when its terms embrace *all* the observations, in which case it is then equivalent to, but not necessarily identical with, some designated mathematical line.

[2] Ryd. V. H. On the computation of meteorological observations.

The daily mean of even 40 years of observations is, after all, backed by only the paltry number of 40 observations, since each daily value can have only unit weight. The annual mathematical curve computed from the daily values for 40 years, however, is backed by nearly 15,000 observations, and each daily value calculated by the equation is, in reality, supported by thousands of adjacent observations, a result unattainable in any other way without centuries of observations.

In the present case both theory and experience show that the Fourier series converges with extreme rapidity. For a large section of the northeastern part of the United States a single or fundamental harmonic term is found entirely adequate to represent the observations for more than 100 years, while elsewhere harmonics of higher order than the second are quite unnecessary; indeed, from certain points of view their introduction tends to mar a good result rather than improve a poor one.

Finally, it may be stated in behalf of the mathematical methods that the least square calculation of the coefficients of the Fourier series is much more easily accomplished than for any other form of equation with a like or even less number of unknown coefficients to evaluate. Much of the prevalent aversion to the use of strictly mathematical methods of combining observations would be largely dissipated if investigators once fully learned how really simple and mechanical many of the processes are or may be made to be. In this problem mechanical devices for effecting the harmonic analysis (and synthesis, as both are involved) may of course be used, but it seems probable the arithmetical method is simpler and better since it handles the numerical data directly and obviates all curve drawing and readily yields final results in the numerical form actually required.

THE DATA AND THEIR ARRANGEMENT FOR ANALYSIS.

No attempt will be made to give in this paper the simple arithmetical steps necessary in the calculation of the few harmonic coefficients required, although the importance is recognized of setting out these steps clearly for the benefit of beginners, and it is hoped this may be done in another connection. It is desired, however, to call attention to some features of this analysis which appear to be original and sufficiently advantageous to merit wider adoption in other meteorological studies.

It is well known that the harmonic analysis of data is most readily effected when the observations cover one complete cycle and fall at exactly equal intervals of time or points of separation; also that the number of intervals are exactly divisible by 4, and with some additional advantages if divisible by 8. Unfortunately the only factors of the common year which may be considered are 5, 73.

Dove's 73 intervals of 5 days or the pentad is fine for some purposes, but useless in the harmonic analysis, because 73 is not divisible by 4 or 8. Gen. R. Strachey[3] used this period, however, by progressively merging one pentad into the 72 remaining ones, thus satisfying the arithmetical demands, but the merging process is tedious, and, at best, the result is a makeshift. The common monthly intervals are of unequal length, and, therefore, unsatisfactory, although frequently employed. The interval, moreover, is too long for the best results. The familiar decade interval would be of great convenience if of rigorously equal length, but as actually used the lengths are indiscriminately unequal by a large propor-

[3] Strachey, Lieut. Gen. R. On the computation of certain harmonic components. Proceedings Royal Soc., Vol. XLII, 1887, p. 61.

tionate part and if made rigorously 10 days long the sub-unit is far from being an aliquot part of the year.

THE WEEK IS THE BEST SUB-UNIT.

Meteorologists have seemingly been slow to recognize the superior advantages of the week as a convenient sub-unit for the discussion of annual meteorological data. Its greater length over the pentad is a real advantage and its shortness compared with the month or the decade is also greatly in its favor. Finally, its crowning superiority for general purposes, including the harmonic analysis, is found in the ease with which the one extra day over 52 weeks in a year may be merged into some one week, and thus realize exactly 52 subunits of the cycle and satisfy practically and easily all requirements. It may be noticed that in this case the mathematical week has a length of $7\frac{1}{52}$ days. If the count begins with January 1, the fraction of a day accumulates progressively and becomes one-half of a day after 26 weeks, which falls about June 2, at about which time the one extra day of the year may be absorbed by including eight days instead of seven in a single week. However, for the treatment of temperature data, it is believed to be still better to merge the extra day into a week about the *middle of July* because the temperatures are then more nearly stationary and the arithmetical effect of including or excluding an extra day in one week is most inconsequential. After a mature consideration of the whole question, I have adopted the following standard sequence of exactly 52 week like sub-units of the year and believe that meteorologists will find this subdivision of the annual cycle well suited to many purposes.

The cycle of weeks may, of course, start at any desired date, but it is considered highly convenient, if not essential, that the cycle divide at 0 hours January 1; that is, January 1 to 7 must be one week of the standard series but need not necessarily be counted as the zero week, since for different purposes it may be desirable to start the count with another week, as, for example, in studying precipitation and drainage, to start the week falling about the 1st of November.

July 16 is chosen as the extra day and is merged into week July 9 to 16. In leap years February 29 is similarly merged in the week February 26 to March 4. In full quadrennial years February 29 must of course be given only one-fourth the weight when combined with data for other dates. Without appreciable error, the mean of seven daily values may be considered as the average value for the mid date of the week. Accordingly, the weeks are designated by the date of the midday as January 1 to 7 designated January 4, July 2 to 8 as July 5, etc.

End correction.—Every complete cycle must, of course, close upon itself. In the year, for example, the fifty-third weekly value of temperature must be identical with the first weekly value. As a rule, the observational data will not satisfy this condition and a correction may be required—here called an end correction—to allow for the failure of the data to close properly as shown by the difference between the first and the fifty-third value of the cycle. This correction in harmonics is mathematically similar to the correction required in problems of triangulation in order to make the sum of the observed angles of a triangle equal to 180°. The whole matter is mentioned at this point simply to emphasize the advantage in temperature studies of starting the count of weeks in *midsummer.* The week of July 5 is preferred and recommended because irregular temperature fluctuations are then much smaller than in December and January, and the end correction will frequently be found so small as to be quite negligible and unnecessary.

In accordance with the foregoing, therefore, the standard schedule of weeks with dates and serial numbers for temperature studies is as followa:

Standard schedule of weeks for temperature studies.

Date.	Number.	Date.	Number.	Date.	Number.	Date.	Number.
July 5....	0	Oct. 5....	13	Jan. 4....	26	Apr. 5...	39
July 12[1]..	1	Oct. 12...	14	Jan. 11...	27	Apr. 12..	40
July 20...	2	Oct. 19...	15	Jan. 28...	28	Apr. 19..	41
July 27...	3	Oct. 26...	16	Jan. 25...	29	Apr. 26..	42
Aug. 3....	4	Nov. 2...	17	Feb 1....	30	May 3...	43
Aug. 10...	5	Nov. 9...	18	Feb. 8....	31	May 10..	44
Aug. 17...	6	Nov. 16..	19	Feb. 15...	32	May 17 .	45
Aug. 24...	7	Nov. 23..	20	Feb. 22...	23	May 24 .	46
Aug. 31...	8	Nov. 30..	21	Mar. 1[2]...	34	May 31..	47
Sept. 7....	9	Dec. 7....	22	Mar. 8....	35	June 7...	48
Sept. 14...	10	Dec. 14...	24	Mar. 15...	36	June 14 .	49
Sept. 21...	11	Dec. 21...	24	Mar. 22...	37	June 21..	50
Sept. 28...	12	Dec. 28...	25	Mar. 29...	38	June 28..	51

[1] This week contains 8 days, viz, July 9-16, inclusive, and the sum of the daily data is divided by 8 to form the weekly mean.

[2] This week contains 8 days in leap year. Data for February 29, when combined with data for other days in full quadrennial years, should enter with a weight of ¼ and the weekly sum be divided by 7¼ to form the weekly mean.

While the schedule of weeks designated above has been finally adopted as the best, the daily observations herein employed were originally tabulated strictly on the calendar dates, which necessitated application of the *end correction* for the year beginning with the first week of January. Moreover, the week of July 20, instead of July 5, was first chosen as the zero week, and the harmonic analysis was made entirely on this basis. When it was finally found that July 5 is, for several reasons, the best zero date for general work it was easy to adjust the equations found to this end by an appropriate change in the phase constants ϕ_1 and ϕ_2 which are the only terms affected.

Stations considered.—Two groups of data have been examined. Schott [4] has published daily means of temperature from certain early records prior to the origin of the Weather Bureau. Certain of these have been introduced for comparison with the Weather Bureau records for later dates.

Weekly means reduced from these appear in Table 1, Nos. 1 to 7, inclusive. In Table 2 is also introduced Equation No. 23*a*, which represents a long record of observations at New Haven, Conn., from 1778 to 1865, which were most carefully coordinated and discussed by Messrs. Loomis and Newton [5]. The authors reduce the New Haven observations to monthly means for the whole series, which, after correcting for curvature and inequality of length of the months, were fitted by the following complete harmonic equation:

$$T_x = 49.11 + 22°.92 \sin(x + 263° 38') + 0°.29 \sin(2x + 345° 24') + 0°.45 \sin(3x + 229° 50') + 0°.02 \sin(4x + 150°) + 0°.38 \sin(5x + 54° 31') - 0.08 \cos 6x,$$

in which x is reckoned from January 15

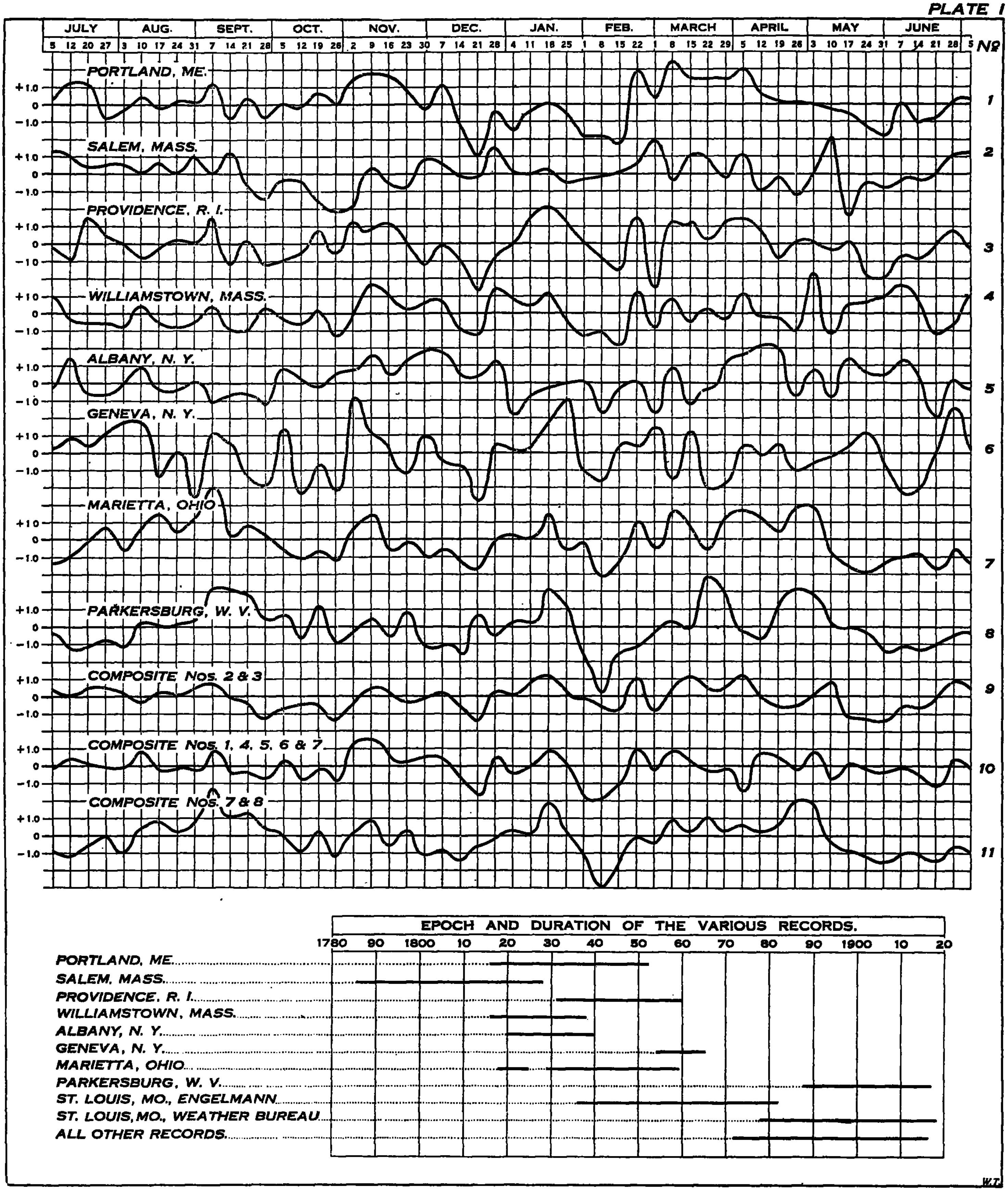
Since this equation contains 12 constants, it fits the 12 monthly observations perfectly. Equation 23a, Table 2, is two terms of the Loomis equation transformed to the cosine function and with abscissas reckoned from July 5.

Neither daily nor weekly means are available for this record, hence no annual diagram of weekly residuals is possible.

[4] Schott. Tables, distribution and variation of atmospheric temperatures in the United States, p. 187.

[5] Loomis, Elias and Newton, H. A. Meteorology of New Haven. Trans. Conn. Acad. Arts and Science, vol. 1, pt. 1, 1866. p. 194.

PLATE I


JULY AUG. SEPT. OCT. NOV. DEC. JAN. FEB. MARCH APRIL MAY JUNE
NO.
PORTLAND, ME.
SALEM, MASS.
PROVIDENCE, R. I.
WILLIAMSTOWN, MASS.
ALBANY, N. Y.
GENEVA, N. Y.
MARIETTA, OHIO
PARKERSBURG, W. V.
COMPOSITE Nos. 2 & 3
COMPOSITE Nos. 1, 4, 5, 6 & 7
COMPOSITE Nos. 7 & 8
+1.0
0
-1.0
EPOCH AND DURATION OF THE VARIOUS RECORDS.
1780 90 1800 10 20 30 40 50 60 70 80 90 1900 10 20
PORTLAND, ME.
SALEM, MASS.
PROVIDENCE, R. I.
WILLIAMSTOWN, MASS.
ALBANY, N. Y.
GENEVA, N. Y.
MARIETTA, OHIO
PARKERSBURG, W. V.
ST. LOUIS, MO., ENGELMANN
ST. LOUIS, MO., WEATHER BUREAU
ALL OTHER RECORDS
W.T.

WEATHER BUREAU STATIONS.

The principal Weather Bureau data analyzed comprised observations of maximum and minimum temperatures from 14 widely distributed stations with continuous record since 1872 or shortly thereafter.

Daily means of the maximum and minimum were formed in four groups for each station, namely, 1872, or the beginning of the record to 1886, 15 years or less in a few cases, designated the first period; 1887 to 1901, 15 years, second period; and from 1902 to 1916, 15 years, third period. Finally, the means were formed for the whole period of the record, 1872 to 1916, 45 years.

the table gives the values only to the nearest tenth of a degree.

The constants of the harmonic equation were computed for each group of data. Generally but a single term was used, especially for the old records and the 15-year groups. The constants for the harmonic of the second order were also determined for such of the long records as needed it. A graph was then prepared showing the weekly residuals or differences between the observed weekly temperatures and those calculated by the harmonic equation. These are reproduced after reduction in Plates I to IV, and clearly show the marked irregularities in the records of the annual march of temperature. Figure 1, prepared for another study,

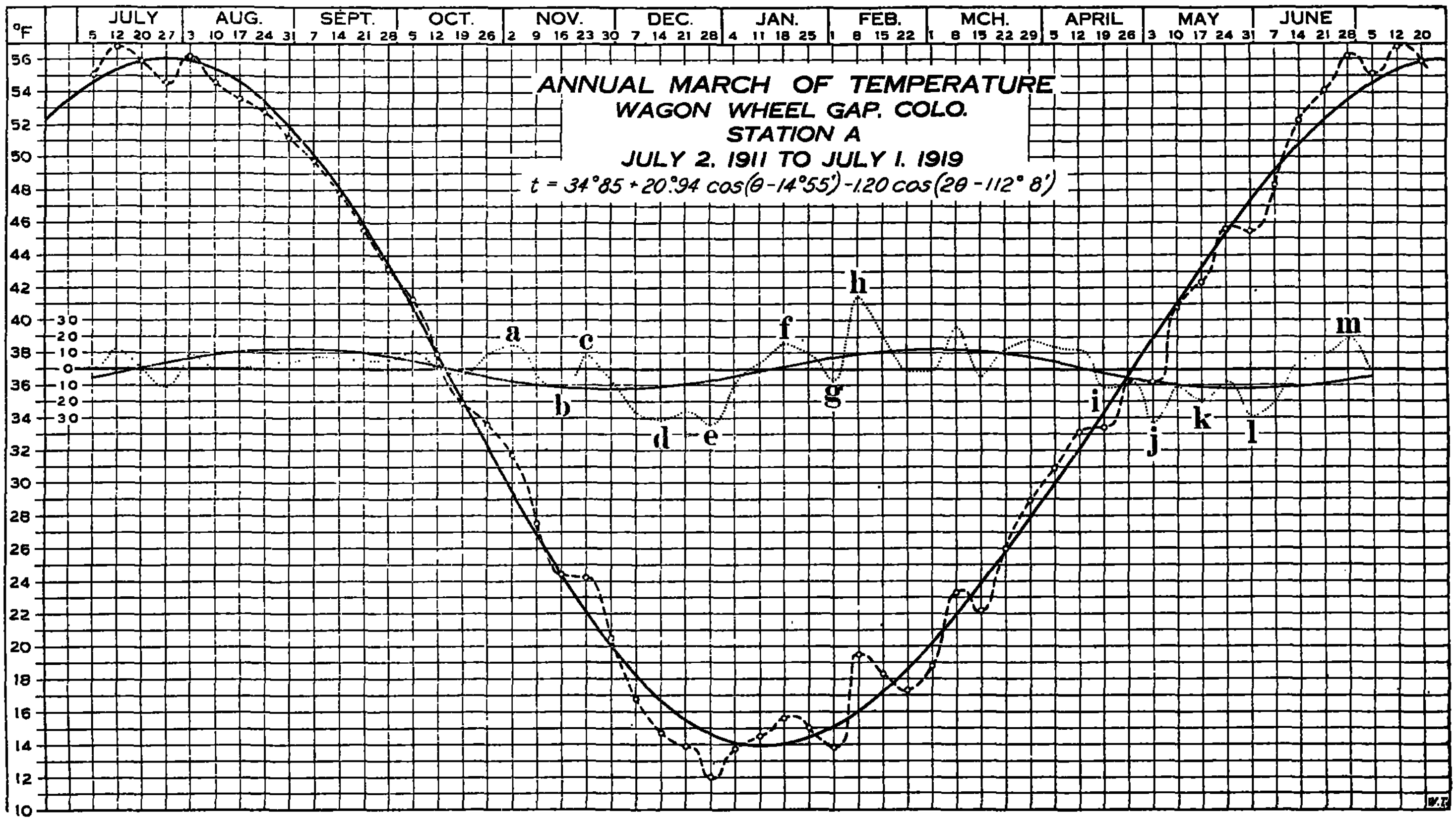


FIG. 1.

shows the complete picture of the present method of analyzing annual temperatures.

Much assistance in the preparation of the data for harmonic analysis was rendered by Mr. Frederic A. Young, of the Climatological Division.

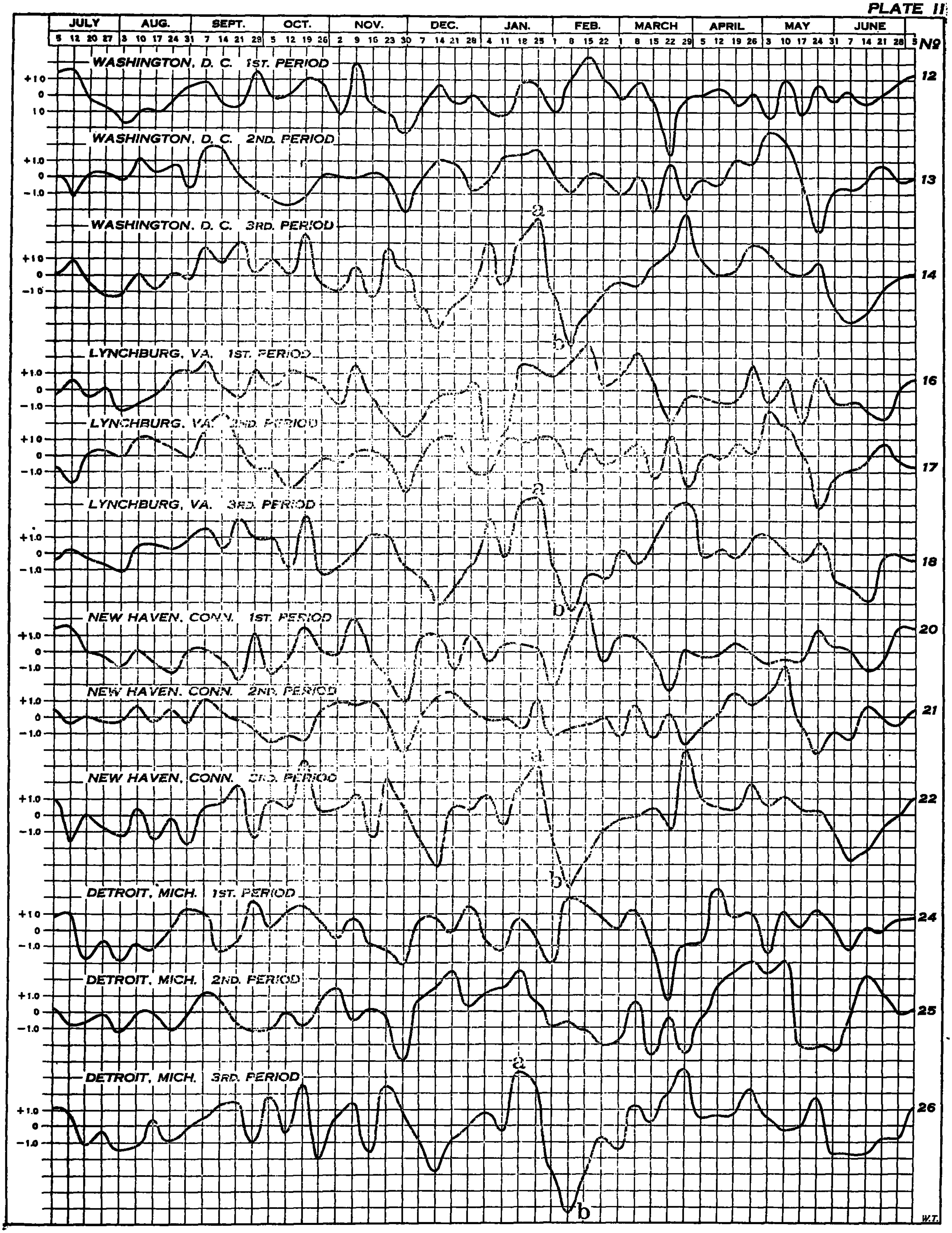
Treatment of data.—Each group of data was reduced to 52 standard weekly means, and, when necessary, as in the case of means from maximum and minimum readings, the values were adjusted to a closer approximation to the means of 24 hourly observations by the application of corrections derived from tables of monthly values published in Bulletin S, United States Weather Bureau.

Table I contains the fully adjusted values of the weekly means for all the stations considered, but in the case of the Weather Bureau stations the means for the whole period of the record only appear. In the original work and computations all values were carried to hundredths of a degree. This order of accuracy is justified because the problem in hand deals with differences and residuals of temperatures, which may be carried to hundredths of a degree even though the absolute value of the temperature may be in some doubt. However, to save space,

The residuals in some of the 15-year groups would have been materially improved by the introduction of the second-order term. This, however, was uniformly omitted and included when needed only in the equation for the whole period.

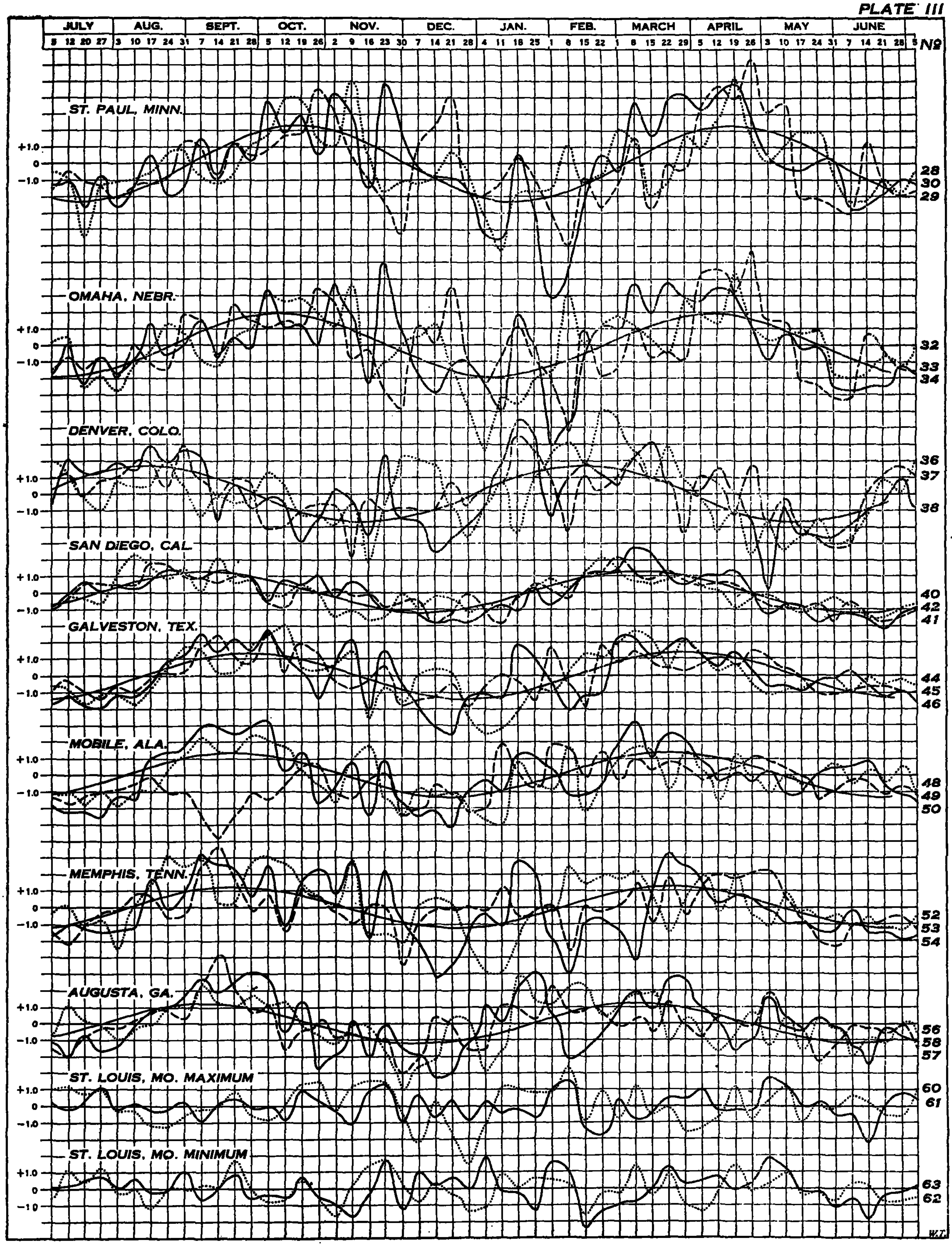
How many terms are needed.—Whether a second term or others of higher order may be needed is not easily determined. It is well known, of course, that every term added to the Fourier series serves to make the sum of the squares of the residuals smaller. While this might be mathematically a good index of need, nevertheless it is fallacious, as such a procedure is not supported by physical considerations. The extreme amplitude of the second term found necessary for any of the data was at St. Paul, 2.27, and while computed for Washington and other long Weather Bureau records, values less than 1.00 have been disregarded in constructing the diagram, because the reality of such harmonics is to be doubted from the following considerations.

PLATE II



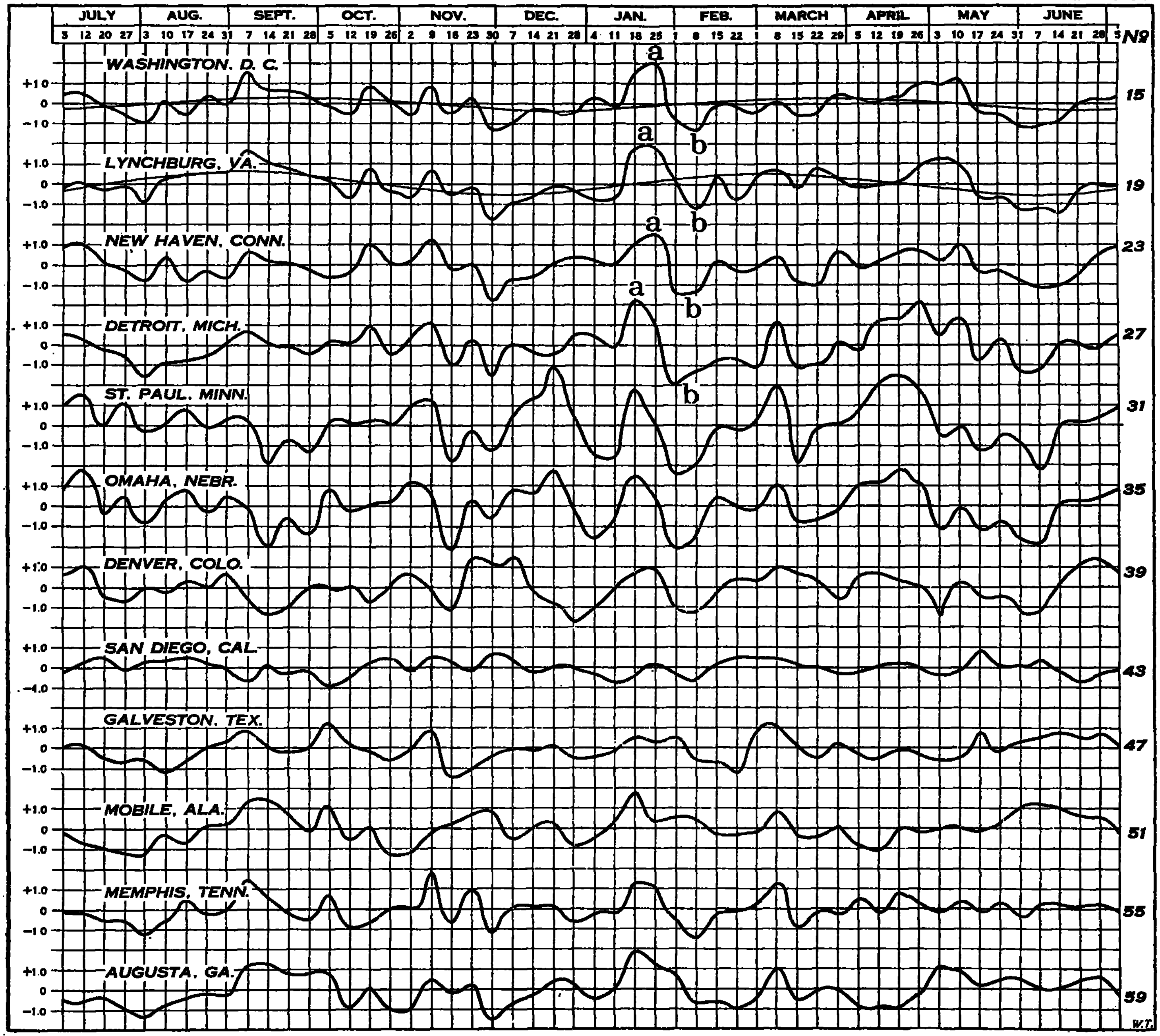
JULY
AUG.
SEPT.
OCT.
NOV.
DEC.
JAN.
FEB.
MARCH
APRIL
MAY
JUNE
No
WASHINGTON, D. C. 1ST. PERIOD
12
WASHINGTON, D. C. 2ND. PERIOD
13
WASHINGTON, D. C. 3RD. PERIOD
14
LYNCHBURG, VA. 1ST. PERIOD
16
LYNCHBURG, VA. 2ND. PERIOD
17
LYNCHBURG, VA. 3RD. PERIOD
18
NEW HAVEN, CONN. 1ST. PERIOD
20
NEW HAVEN, CONN. 2ND. PERIOD
21
NEW HAVEN, CONN. 3RD. PERIOD
22
DETROIT, MICH. 1ST. PERIOD
24
DETROIT, MICH. 2ND. PERIOD
25
DETROIT, MICH. 3RD. PERIOD
26
+1.0
0
-1.0
a
b
W.T.

PLATE III



JULY
AUG.
SEPT.
OCT.
NOV.
DEC.
JAN.
FEB.
MARCH
APRIL
MAY
JUNE
NO
ST. PAUL, MINN.
28
30
29
OMAHA, NEBR.
32
33
34
DENVER, COLO.
36
37
38
SAN DIEGO, CAL.
40
42
41
GALVESTON, TEX.
44
45
46
MOBILE, ALA.
48
49
50
MEMPHIS, TENN.
52
53
54
AUGUSTA, GA.
56
58
57
ST. LOUIS, MO. MAXIMUM
60
61
ST. LOUIS, MO. MINIMUM
63
62
+1.0
0
-1.0
W.T.

PLATE IV



The diagrams clearly show that, if an additional term is systematically called for, the residuals will be disposed harmonically with reference to the axis, as, for example, in the case of the superposed curves of Plate III showing the second harmonic in place, whereas the curves on Plate IV just as clearly show the absence of any systematic disposition of the residuals or the need of additional harmonic terms. Doubts on this point are easily removed if one will only himself compute a supposed coefficient of higher order and critically observe how the value thereof in many cases depends almost entirely on one or a very few favorably located residuals of large magnitude. In such cases the fit of the curve to the data is improved at one point but largely at the expense of the fit at many other places, a procedure not justified by the physics of the problem, even though the sum of the squares of the residuals may be reduced. This process of impairing the fit at one place in order to improve it at another in a certain sense tends to force the residuals into a harmonic arrangement.

The best guide I have found is to draw the second, third, and any higher harmonics desired on tracing paper and to the same scale as the diagrams. The amplitude may be anything desired. (The second harmonic for Denver was conveniently copied for this purpose.) The superposition of one of these tracings over the curve of residuals shows in a very convincing way how the residuals at one portion of the year will support and at other times oppose a harmonic of the period chosen, or possibly, if needed, systematically support it, as in the case of the second harmonics shown. In some cases, a half or a third of a year may support a higher harmonic, but the remainder of the year will quite as systematically oppose it.

The writer is firmly convinced this study demonstrates that the Fourier series, as applied to annual temperature data, converges only to the second term. After that it is not a question of convergence but simply a hit-and-miss fit of a purely arbitrary wave form to a relatively small number of residuals. A few large residuals anywhere throughout the year will give a finite value to almost any harmonic.

Important caution.—The arithmetic of harmonics is really fascinating and alluring, and the earnest student in search of physical truths must not allow himself to be deluded by the finite value of harmonics of the higher orders.

TABLE 1.—*Weekly means of temperature.*

Date middle period of each week.	Portland, Me., 1816 to 1852.	Salem, Mass., 1786 to 1828.	Providence, R. I., 1831 to 1860.	Williamstown, Mass., 1816 to 1838.	Albany, N. Y., 1820 to 1840.	Geneva, N. Y., 1854 to 1865.	Marietta, Ohio, 1818 to 1823, 1829 to 1859.	Parkersburg, W. Va., 1888 to 1917.	Washington, D. C., 1872 to 1916.	Lynchburg, Va., 1874 to 1916.	New Haven, Conn., 1873 to 1916.	Detroit, Mich., 1872 to 1916.	St. Paul, Minn., 1872 to 1916.	Omaha, Nebr., 1872 to 1916.	Denver, Colo., 1872 to 1916.	San Diego, Calif., 1875 to 1916.	Galveston, Tex., 1872 to 1916.	Mobile, Ala., 1872 to 1916.	Memphis, Tenn., 1872 to 1916.	Augusta, Ga., 1874 to 1916.	St. Louis, Mo., Engelmann-maximum, 1836 to 1882.	St. Louis, Mo., maximum, 1878 to 1918.	St. Louis, Mo., Engelmann-minimum, 1836 to 1882.	St. Louis, Mo., minimum, 1878 to 1918.
Graph number.	1	2	3	4	5	6	7	8	15	19	23	27	31	35	39	43	47	51	55	59	60	61	62	63
July 5	65.5	72.8	69.3	70.4	72.1	69.3	71.9	75.2	76.5	76.6	71.0	72.2	71.3	76.2	71.5	65.4	83.0	80.6	80.0	80.0	87.4	87.1	69.3	70.1
July 12	67.1	73.1	71.3	69.8	71.4	70.7	72.7	74.8	76.9	77.1	71.8	72.7	72.2	77.8	72.7	66.4	83.4	80.5	80.4	80.4	88.4	88.5	70.9	71.0
July 20	67.6	72.9	72.3	69.8	72.6	70.7	75.7	75.1	76.5	77.1	71.4	71.5	70.9	76.0	71.8	67.2	83.2	80.6	80.4	81.0	87.5	87.5	69.8	70.6
July 27	65.7	73.0	71.3	69.7	72.3	71.5	74.2	75.2	76.1	77.0	71.2	72.4	71.9	76.8	71.7	67.1	83.2	80.6	80.4	80.6	87.6	88.2	70.0	71.2
Aug. 3	65.8	72.6	70.5	69.0	72.3	71.8	72.2	74.4	75.2	75.9	70.4	71.0	70.2	75.2	72.1	67.8	83.4	80.4	79.5	80.0	86.6	87.6	68.4	70.0
Aug. 10	66.1	71.5	70.8	69.4	72.5	71.2	72.8	75.1	75.6	76.2	71.1	71.0	69.7	75.6	71.4	68.1	82.6	81.1	79.6	80.1	86.0	87.2	68.9	69.7
Aug. 17	64.5	70.9	68.8	67.2	69.9	67.2	72.4	73.8	71.0	75.3	69.1	70.1	69.5	75.2	70.9	68.5	82.8	80.2	79.8	79.6	85.7	86.9	67.9	69.8
Aug. 24	63.7	69.0	68.0	65.5	68.4	67.4	69.8	72.4	73.4	74.1	68.4	69.0	67.3	73.0	69.3	68.2	82.9	80.2	78.1	78.9	84.8	84.6	66.8	67.0
Aug. 31	62.1	68.3	66.4	61.1	67.0	63.4	69.0	71.0	71.5	72.6	66.7	68.1	66.1	72.0	68.3	67.9	82.4	79.4	77.0	77.6	83.3	84.3	64.9	66.2
Sept. 7	61.3	65.4	66.1	62.7	63.7	65.1	68.8	70.8	71.2	72.0	66.2	66.7	64.0	69.4	65.0	66.9	82.0	79.2	76.8	77.6	82.1	83.0	64.6	65.4
Sept. 14	57.3	64.4	61.5	59.3	61.8	62.4	61.0	68.8	68.3	69.4	64.0	64.0	59.7	65.3	61.8	67.2	80.1	77.9	74.1	75.8	78.4	80.3	60.6	61.9
Sept. 21	56.1	60.2	60.4	56.8	59.2	58.2	62.3	66.2	66.0	67.0	61.6	61.4	58.2	61.1	60.4	66.3	78.4	75.7	71.3	73.4	75.4	78.2	57.3	59.8
Sept. 28	52.6	56.9	56.8	55.4	56.0	55.7	59.5	62.5	63.4	64.4	59.1	58.7	51.9	60.6	57.7	65.6	76.9	73.9	68.6	71.3	73.3	74.8	54.6	57.1
Oct. 5	50.7	55.3	54.4	52.0	55.3	56.0	56.1	60.0	60.4	61.8	56.2	56.2	53.0	59.6	51.9	64.0	76.4	72.2	67.3	69.1	71.4	73.8	52.8	55.7
Oct. 12	47.9	53.1	52.2	48.7	51.7	49.6	53.0	56.1	57.4	58.6	53.9	53.3	49.9	55.3	52.0	63.3	73.6	68.6	63.2	65.0	67.1	70.1	48.8	51.6
Oct. 19	45.8	48.4	50.8	46.4	48.3	48.5	50.8	55.2	56.1	57.4	52.7	51.2	46.5	52.1	48.4	63.0	71.2	66.9	61.7	63.5	62.8	66.2	44.7	48.2
Oct. 26	42.3	45.0	46.8	42.1	46.2	44.2	47.7	50.4	52.8	53.9	49.1	46.9	42.6	48.8	46.4	62.2	68.6	63.4	57.9	60.0	60.7	62.2	43.3	44.5
Nov. 2	41.0	42.6	45.8	40.5	43.4	46.7	47.2	48.4	49.4	51.2	46.6	44.7	39.8	46.2	44.1	60.6	67.0	61.3	56.1	57.7	57.6	60.8	41.7	43.4
Nov. 9	38.5	41.8	42.7	39.4	41.4	41.9	45.4	46.6	48.3	50.1	45.0	42.6	36.4	42.0	40.8	60.1	65.9	60.7	55.2	56.9	54.1	57.9	39.1	41.8
Nov. 16	35.7	38.4	40.5	35.9	37.6	38.2	41.1	43.2	44.5	46.7	41.0	37.7	29.8	35.8	37.3	58.9	61.6	58.6	50.3	54.1	49.0	52.5	35.1	36.1
Nov. 23	32.2	35.5	36.9	32.6	36.0	34.6	39.4	42.2	42.8	45.0	38.8	36.2	27.7	34.9	37.6	57.5	62.1	57.2	49.6	52.5	44.1	51.0	31.6	35.5
Nov. 30	28.8	31.8	33.4	30.6	34.2	34.3	36.6	38.2	39.1	41.4	34.9	32.1	23.5	30.9	35.6	57.3	58.9	53.7	45.4	49.0	43.8	46.8	30.4	32.2
Dec. 7	28.0	32.4	32.4	28.5	32.0	30.7	35.2	36.4	37.7	40.5	33.8	31.3	22.0	29.4	34.2	56.3	57.7	52.8	44.8	48.3	39.4	45.4	27.8	30.3
Dec. 14	23.8	29.8	29.7	24.9	28.8	31.4	33.1	34.2	36.5	39.4	32.2	29.0	20.2	26.9	31.1	55.1	56.3	52.1	43.0	47.5	35.8	41.9	25.2	27.6
Dec. 21	20.2	28.2	26.4	22.2	27.3	24.9	31.3	35.0	35.0	38.6	31.1	28.0	19.4	25.9	29.6	54.8	55.4	51.4	42.0	47.2	36.5	41.4	24.2	27.8
Dec. 28	21.4	28.6	26.9	21.5	26.8	26.7	32.2	32.8	33.8	37.5	30.1	26.6	15.1	22.4	27.8	54.3	53.9	49.7	40.3	46.5	37.8	39.3	25.3	24.5
Jan. 4	19.3	26.2	25.7	22.9	23.0	25.2	31.8	32.8	33.7	36.2	29.0	25.3	11.9	19.9	28.1	53.8	53.4	49.6	40.1	45.3	37.1	38.5	24.0	23.4
Jan. 11	19.7	25.5	27.5	22.0	23.5	24.7	31.3	32.1	32.8	35.9	28.0	24.2	10.9	20.1	29.2	53.4	53.3	50.3	40.0	45.9	36.9	37.6	24.3	21.8
Jan. 18	19.6	25.4	27.6	22.4	23.8	25.7	32.6	33.9	34.3	38.2	28.5	26.0	14.0	22.2	29.8	53.6	54.1	52.0	41.7	47.8	37.3	40.8	24.4	24.6
Jan. 25	19.4	24.6	26.7	20.8	22.8	26.0	30.8	33.1	31.8	38.5	29.0	24.7	12.6	21.5	30.4	54.1	54.2	51.0	41.9	47.7	39.7	40.4	25.7	24.4
Feb. 1	17.8	25.3	25.9	19.5	21.3	23.0	31.8	30.7	32.5	37.3	26.3	22.1	10.9	20.0	29.2	53.9	55.0	51.9	41.2	48.0	37.7	38.3	23.7	21.6
Feb. 8	18.5	26.0	25.5	20.5	24.0	23.2	20.7	29.3	32.6	36.7	26.8	23.3	12.8	21.9	29.8	53.9	54.6	52.8	41.2	47.5	41.0	39.3	26.0	23.6
Feb. 15	18.9	27.3	25.8	22.0	26.5	26.1	32.8	32.8	35.0	39.2	29.0	24.8	16.3	25.6	31.8	55.1	55.4	53.2	43.8	48.7	41.5	43.0	27.4	25.5
Feb. 22	24.3	29.3	30.0	26.5	28.5	27.3	36.5	34.5	36.2	39.5	29.8	26.4	18.7	27.4	33.7	55.7	57.6	54.4	45.5	49.7	45.6	44.0	30.7	27.3
Mar. 1	24.3	32.1	26.6	26.2	28.6	29.9	36.6	37.0	37.4	42.0	31.4	27.6	21.8	29.8	35.1	56.1	59.6	56.0	47.6	51.8	46.4	46.7	30.8	30.0
Mar. 8	28.2	31.8	32.9	29.9	33.2	28.8	40.6	40.4	39.8	44.1	33.6	31.6	26.6	33.7	37.3	56.4	61.3	58.5	50.6	54.8	49.4	50.2	34.0	33.6
Mar. 15	29.2	35.4	35.0	30.8	33.5	33.6	41.8	41.2	41.1	45.3	34.4	31.6	26.2	34.9	38.6	56.6	61.9	58.9	50.6	55.0	50.8	52.8	34.3	35.1
Mar. 22	31.5	37.5	36.4	34.1	36.8	32.7	42.8	46.4	43.5	48.2	36.3	34.2	31.3	38.2	40.0	56.9	63.0	60.5	53.4	57.2	53.7	56.1	36.3	37.9
Mar. 29	34.0	39.0	39.7	36.3	41.1	35.3	47.0	47.9	46.8	50.2	40.2	37.8	35.2	41.9	41.2	57.1	65.3	62.7	55.6	59.1	57.1	57.7	39.7	40.8
Apr. 5	37.2	43.0	42.5	40.5	44.4	40.1	50.0	48.2	49.1	52.0	41.8	40.2	39.5	46.5	44.4	57.6	66.5	63.5	58.6	60.1	61.7	60.6	43.8	42.7
Apr. 12	38.5	43.8	44.3	42.0	47.8	42.3	52.1	50.5	51.6	54.5	44.7	44.6	44.4	49.9	46.6	58.3	67.8	64.9	60.1	62.0	62.5	63.4	44.7	45.9
Apr. 19	40.7	47.4	45.5	45.0	50.6	45.7	53.8	55.0	54.5	57.1	47.8	47.6	48.6	53.8	48.5	58.9	69.9	67.6	63.4	63.8	66.3	67.2	47.6	49.2
Apr. 26	43.6	49.3	49.2	47.2	50.9	46.9	57.8	58.7	57.7	60.3	50.5	51.3	51.2	56.3	50.6	59.1	71.2	69.1	65.0	66.3	69.7	70.3	50.8	52.3
May 3	46.4	53.4	51.9	53.3	55.3	50.2	60.4	61.1	60.4	63.1	52.7	52.7	52.3	57.1	51.4	59.3	72.4	70.8	67.1	69.5	70.2	71.6	51.9	53.5
May 10	48.8	54.0	54.1	52.7	56.5	53.3	60.0	62.0	63.2	65.2	56.1	56.4	52.6	61.0	55.4	60.1	74.0	72.3	69.2	71.0	73.6	74.3	54.5	56.1
May 17	51.3	56.4	57.2	57.2	61.3	56.6	61.6	61.2	64.0	65.8	57.2	57.2	57.4	62.5	57.0	60.6	75.5	73.6	70.6	72.0	75.6	75.7	56.4	57.4
May 24	53.2	60.8	57.7	59.9	63.1	60.0	63.5	66.2	66.2	67.9	59.7	60.8	60.6	65.4	59.4	61.5	77.1	75.4	72.8	73.9	78.4	77.9	59.6	60.2
May 31	54.9	62.9	59.8	62.6	65.3	60.3	65.9	67.2	67.8	69.3	61.6	61.7	62.5	66.5	60.8	62.0	78.7	77.4	73.8	75.5	80.0	78.0	61.7	61.0
June 7	59.0	65.4	63.2	65.4	68.4	60.9	68.0	69.5	69.7	71.0	63.3	64.2	63.3	68.4	63.2	63.0	80.0	78.5	76.0	76.4	81.0	81.9	62.6	64.2
June 14	59.9	67.3	64.9	66.4	69.3	63.1	68.6	71.0	71.6	72.3	65.3	67.4	67.0	71.9	66.5	63.2	81.2	79.4	77.3	77.6	82.5	84.2	64.4	66.5
June 21	61.9	69.3	64.4	66.0	68.4	67.0	70.1	72.6	73.9	74.7	67.3	69.1	68.6	73.7	69.2	63.4	81.9	79.9	78.4	79.2	84.5	85.2	66.1	67.7
June 28	62.8	71.6	69.4	67.9	71.7	70.7	72.2	74.2	75.3	75.9	69.6	70.3	69.9	74.9	70.8	64.5	82.8	80.6	79.1	80.3	86.9	85.7	69.0	69.0

TABLE 2.—*Stations and constants of the equation of the annual march of temperature.*

[$T=A_0+a_1 \cos(\theta-\phi_1)+a_2 \cos(2\theta-\phi_2)$. θ is reckoned from July 5. Numbers correspond to Table 1 and diagrams.]

Station.	No.	Period.	An'l. mean temp.	a_1.	ϕ_1.		a_2.	ϕ_2.	
			°F.		°	′		°	′
Portland, Me. 1816–1852.	1	....	42.96	23.53	19	21	......	...	..
Salem, Mass. 1786–1828.	2	....	48.75	23.70	16	44	......	...	..
Providence, R. I. 1831–1860.	3	....	48.19	22.68	18	31	......	...	..
Williamstown, Mass. 1816–1838.	4	....	45.74	24.53	15	9	......	...	..
Albany, N. Y. 1820–1840.	5	....	48.61	24.54	13	55	......	...	..
Geneva, N. Y. 1854–1865.	6	....	47.14	23.26	18	49	......	...	..
Marietta, Ohio 1818–1859.	7	....	52.40	21.36	11	18	......	...	..
Parkersburg, W. Va. 1888–1917.	8	....	53.97	22.21	14	7	......	...	..
Washington, D. C.	12	First....	54.51	22.72	16	25	......	...	..
	13	Second..	54.76	21.79	15	14	......	...	..
	14	Third...	54.80	21.62	14	28	......	...	..
	15	All......	54.69	22.04	15	20	0.32	149	33
Lynchburg, Va.	16	First....	57.16	20.59	13	8	......	...	..
	17	Second..	56.86	20.44	13	36	......	...	..
	18	Third...	56.68	20.16	12	31	......	...	..
	19	All......	56.88	20.42	13	29	0.55	117	13
New Haven, Conn.	20	First....	49.47	22.51	20	38	......	...	..
	21	Second..	49.56	22.39	19	14	......	...	..
	22	Third...	50.19	22.14	19	18	......	...	..
	23	All......	49.37	22.02	19	43	0.07	184	18
New Haven, Conn., Loomis. 1778–1865.	23a	....	49.11	22.92	17	43	0.29	127	17
Detroit, Mich.	24	First....	48.02	24.42	19	2	......	...	..
	25	Second..	48.36	24.70	17	40	......	...	..
	26	Third...	48.26	24.44	18	25	......	...	..
	27	All......	48.22	24.47	18	26	0.39	235	12
St. Paul, Minn.	28	First....	43.60	29.59	12	8	......	...	..
	29	Second..	43.62	29.87	13	29	......	...	..
	30	Third...	44.45	28.81	15	6	......	...	..
	31	All......	43.83	29.39	14	6	2.27	204	59
Omaha, Nebr.	32	First....	49.45	28.61	14	26	......	...	..
	33	Second..	50.40	27.69	12	53	......	...	..
	34	Third...	50.85	27.23	14	44	......	...	..
	35	All......	50.23	27.78	13	55	1.87	188	39
Denver, Colo.	36	First....	49.13	22.97	14	12	......	...	..
	37	Second..	49.98	21.49	14	52	......	..	...
	38	Third...	49.87	20.82	15	55	......	...	..
	39	All......	49.66	21.53	14	59	1.69	80	1
Wagon Wheel Gap, Colo. 1911–1919.	39a	....	34.85	20 94	14	55	1.20	112	8
San Diego, Calif.	40	First....	60.37	7.25	25	17	......	...	..
	41	Second..	60.67	6.56	33	47	......	...	..
	42	Third...	60.56	6.38	36	26	......	...	..
	43	All......	60.54	6.69	32	2	1.25	127	32
Galveston, Tex.	44	First....	70.05	15.75	14	49	......	...	..
	45	Second..	69.47	14.69	16	44	......	....	.
	46	Third...	69.36	14.78	17	31	......	...	..
	47	All......	69.63	15.07	17	0	1.38	165	8
Memphis, Tenn.	48	First....	60.99	21.03	11	54	......	...	..
	49	Second..	61.58	19.79	11	24	......	...	..
	50	Third...	61.33	19.87	13	21	......	...	...
	51	All......	61.30	20.27	12	11	1.21	152	24
Mobile, Ala.	52	First....	66.85	16.54	12	57	......	...	..
	53	Second..	66.02	15.22	11	38	......	...	..
	54	Third...	66.91	15.37	12	28	......	...	..
	55	All......	66.59	15.72	12	22	1.34	149	48
Augusta, Ga.	56	First....	64.24	17.55	12	18	......	...	..
	57	Second..	63.62	17.68	10	49	......	...	..
	58	Third...	63.67	17.65	10	53	......	...	..
	59	All......	63.84	17.69	11	48	1.18	128	22
St. Louis, Mo., E. max.	60	....	63.33	25.69	10	37	2.26	150	36
St. Louis, Mo., W. B. max.	61	....	64.79	24.73	13	59	2.02	174	58
St. Louis, Mo., E. min.	62	....	46.89	23.04	12	32	1.12	114	49
St. Louis, Mo., W. B. min.	63	....	47.72	23.75	14	52	1.10	181	32

E.=Engelmann record. W. B.=Weather Bureau record.

The terms of the Fourier series are always positive and additive. The coefficients (or amplitudes) of all harmonics will be finite *as long as any residuals whatever remain.* Critical scrutiny of the numerical summations of data which enter into the harmonic constants and tests by the superposition of tracing paper curves are the surest guides.

While this paper clearly shows how well the harmonic equation of the second order represents the *annual* march of temperature, it must be recognized the demonstration stops right there. It is just as easy to show that the Fourier series is quite *unfit* to represent the *diurnal* march of temperature, although frequently used for this purpose.

Table 2 contains the constants of the harmonic equations, and the diagrams in Plates I, II, III, and IV, bearing corresponding numbers exhibit the weekly residuals (observed minus calculated) derived therefrom.

DISCUSSION OF EQUATIONS AND RESIDUALS.

A critical examination and comparison of the numerical values in Table II can not fail to impress any one with the great constancy of the values:

A_0, which is the annual mean temperature.

a_1, which is very nearly half the annual range of temperature, and

ϕ_1, which fixes very largely the time of the summer maximum (as also the winter minimum). The values of a_2 and ϕ_2 show much greater variations, but the significance of these also is important in particular cases. We should naturally expect to find but little variation in the value of the *annual mean* temperature when derived from 15 or 45 years of records, but when we compare the Weather Bureau records for New Haven, for example, with the Loomis record No. 23a we may well be gratified and surprised to find no greater differences. These several results are still erroneous and should be expected to differ to some extent, simply because of instrumental errors and the differing influences of exposure and the means employed in each case to deduce a true mean daily temperature from observations of a maximum and minimum, or a few readings a day with occasional breaks which mar the continuity of old records especially.

How basic climatic characteristics, such as the relatively constant annual range, stand out in spite of all these and other accidental disturbing effects is shown by the small differences in the values of a_1, whether deduced from one of the 15-year groups or from a record at a near-by station made a hundred years earlier, as, for example, Salem, Mass., 1786 to 1828, a_1 = 23.70, and New Haven, 1873–1916, a_1 = 22.02, or Marietta, Ohio, 1818–1859 a_1 = 21.36, and the Weather Bureau record at Parkersburg, near by, 1888 to 1917, a_1 = 22.21. The Loomis equation for the New Haven record, 1778 to 1865, Equation 23a, Table 2, and the later Weather Bureau record for the same station, No. 23, are striking cases of close agreement. The same order of constancy is found in the approximate dates of highest and lowest temperatures as indicated by the values of ϕ_1. These dates are of course slightly modified by the values of the second harmonic, and just what the actual dates are does not concern us much in this study, but we may make the observation in passing that the data in Schott's early records seem to indicate a slightly earlier date of maximum and minimum than now prevails. A more critical examination or analysis of the data for the whole century is necessary, however, to establish the real facts.

Just one additional point will be mentioned resulting from the somewhat different treatment given the St. Louis record. A very excellent record of daily values of maximum and minimum observations from 1836 to 1888 was discussed and published by Englemann.[6] These, supplemented by partly overlapping and subsequent records at the Weather Bureau station, have been harmonically analyzed, keeping the maximum and minimum values in separate groups. The important feature which seems to stand out is that the second harmonic is much smaller in the march of minimum than in that of the maximum, which is greater than for the mean temperatures. More data must be studied, however, to get at the

[6] Trans. St. Louis Acad. Sci. IV, p. 496–508. 1878–1886.

real physical facts back of this indication. The feature seems to be a measure of the lag exhibited by the temperature of a place in responding to the seasonal changes in effective solar radiation.

Having attained its summer maximum, the temperature falls off more slowly all through the autum than the decline in effective insolation, followed by rapid cooling in November, which slows up in December, becomes nil in January, and a reverse lag occurs during the warming up process from February to July. These considerations have a very important bearing upon the claims made by such students as Clayton [7] with reference to the immediate influence he ascribes to small irregular changes in the intensity of solar radiation in effecting noticeable changes in terrestrial weather. A critical study of the lag effects mentioned above should help to evaluate the earth's sensitivity to such influence.

Diagrams.—No verbal discussion can replace a thorough and critical examination of the diagrams which the dubious reader must make for himself if he is skeptical of the conclusions stated later. Accordingly, only a few important considerations will be brought out.

The following features of the diagrams are conspicuous:

(*a*) Without respect to length of record or locality of the station, each annual diagram shows a sequence of a considerable number of irregular pseudo-periodic alternations of crests and hollows, which vary in character from mere inflections to fully developed waves.

(*b*) In most cases the waves are better developed, or of larger amplitude and longer period in the colder months (November to March, inclusive) than in the warmer months.

(*c*) The annual number of waves can not be stated definitely because so much depends upon the inclusion or exclusion of the poorly defined minor waves. However, counting up all the diagrams and including each crest and hollow which may reasonably claim recognition, I find a quite consistent average of about 30 crests and hollows per year, or about 15 full waves. I also estimate that between 5 and 10 per cent of the number are doubtful. This leaves between 13 and 14 major waves per year, or an average interval, crest to crest, of about three and one-half to four weeks. Practically the same average result is reached by a count of the number of changes $\pm$ in the signs of the residuals, but, of course, all do not fall between the crests and hollows as counted.

(*d*) By comparing diagrams for the 15-year periods with the one for the whole period we find in every case the conspicuous crests tend to diminish and disappear. All residuals seem to tend to approach closer and closer to the axis of the diagram as the length of the record increases.

(*e*) While we necessarily assume that the weather or temperature of to-day or of this week is quite independent of the weather or temperature of the same day or week a year ago, or any other year, yet we must recognize that there is a close correlation between the temperature of yesterday, to-day, and to-morrow, and a less close relation between the temperatures of contiguous weeks. Extremes of temperature necessarily occur in spells, often of several days or even weeks duration. Such events impress themselves not only upon human attention and memory, but it is interesting to notice that in a mathematical sense a very like effect is impressed on the averages of the numerical data. A conspicuous crest and hollow, such as *a b* in the Washington record No. 14, signifies a marked alternation of unusual warm weather, culminating in a crest for the week of January 25, followed by an equally pronounced and unusual cold wave, culminating in the week of February 8. The imprint such events make on the memory is lasting and it can be effaced from the numerical average only in two ways, both of which tend to require a very long time:

(1) Obviously the subsequent occurence at or about the same date of another extreme but in the opposite sense more or less completely effaces from the numerical average the imprint of the first event. A chance of this kind, as we know, will occur only rarely. Moreover, it is nearly an equal chance that two continuous extremes will be of the same as of opposite kind and thus tend to perpetuate rather than efface each other.

(2) The other way in which the imprint left by one extreme condition is effaced from the average is simply the prolonged absence subsequently of any other like extremes. The recurrence year after year of average conditions little by little effaces the imprint one or more extremes may have produced originally. These considerations seem equally applicable to any of the conspicuous features present in all the diagrams.

(*f*) If the "January Thaw," the "May Freeze," "Squaw Winter," and "Indian Summer," are concluded to connote some of the 13 or 14 wave features which characterize even the longest records, then all the features claim equal recognition and equally need explanation. The features of weather named above have doubtless impressed themselves upon human attention because of their influence on agriculture or other critical human interests; the 9 or 10 other similar features of the annual cycle have passed unnoticed.

(*g*) The claim may be made that the Ice Saints, for example, are real enough, but the feature is eliminated because it does not fall at closely the same date each year. The mere incident of happening over a range of dates, therefore, causes it to vanish from the final mean. Such a claim is quite refuted by the diagrams, however, because if true the features would be consistently conspicuous in the short or 15-year records, and less conspicuous in the full period. Take New Haven, Conn., for example. The first period, No. 20, Plate II, shows no conspicuous warm spell late in January nor a cold spell early in May. The second period, No. 21, shows the the week of January 25 one degree above the axis, while the first half of May is conspicuously warm. The third period, No. 22, exhibits a well defined January thaw, culminating on the 25th, with May warm throughout, almost. These conspiring and opposing effects impress upon the record for the whole period, No. 23, Plate IV, a semblance of a January thaw.

The Detroit records, Nos. 24, 25, and 26, tell practically an identical story. Indeed, period for period, the Detroit and New Haven records are remarkably alike. We should really expect to, and we do find, likeness in the records of different stations for the same periods. We also find great unlikeness in records of the same or near-by stations for different times.

Critical examination of the records such as outlined in the foregoing leads to but one conclusion. Thirteen or fourteen major alterations of relatively warm and cool weekly temperatures infallibly recur irregularly year after year. These must necessarily conspire and interfere indiscriminately, and great major phenomena, traceable in the original record to a single or a few unusual events which have fortuitously recurred and combined, impress themselves upon the average of the whole record.

[7] H. H. Clayton. Effect of short-period variation of solar radiation on the earth's atmosphere, Smithsonian Misc. Colls., vol. 68, no. 3, 1917, 18 pp.

Note in the New Haven and Detroit records how the striking crests *a b* in the third period still persist in the 45-year means.

It is proper to consider at this point what would have been the outcome of this investigation if it had been feasible to use, say, 72 or 24, instead of 52 week like subunits of the cycle. It seems easy to imagine that the number and distribution of the conspicuous features would be quite different. In other words, the detailed features would depend upon the method of analysis and not have a real existence.

Each striking feature on a long record is, therefore, no evidence of the persistent recurrence of peculiar irregularities, but is simply the residual scar or imprint of some unusual event, or a few which have been fortuitously combined at about the time in question. Time will inevitably efface these, but a very long time is necessary to reduce the curve to even the semblance of smoothness and simplicity. Little is gained by combining the results from many stations unless widely separated or for different periods, because all are likely to be similarly impressed by the same major irregularities. If widely separated, the averages may be smoother, but individual characteristics peculiar to a locality are very likely to be lost. The composite records Nos. 9, 10, and 11, are examples of this treatment.

Referring to the final conclusions stated in the synopsis, it is believed the foregoing studies clear away grave uncertainties and conflicting opinions concerning the annual cycle of temperature and we are now in a position to produce daily, weekly, or other normal values of any kind desired. These are much needed for the purpose of carrying out interesting and important studies which are in contemplation of the frequency distribution and standard deviations of temperature data, including more serious investigations of the cause and effect relations between terrestrial temperatures and cosmic and other influences.

LITERATURE CONCERNING SUPPOSED RECURRENT IRREGULARITIES IN THE ANNUAL MARCH OF TEMPERATURE.

By C. FITZHUGH TALMAN.

[Dated: Weather Bureau Library, Washington, May, 1919.]

The belief that periods of unseasonable heat and cold tend to recur at or about the same time from year to year has prevailed over a great part of the world for many centuries and has been the subject of extensive scientific investigation. Among the most widely recognized periods of this character are the following:

1. *A mild period in January, the "January thaw."*—This period is popularly looked for in America, especially in New England, but apparently not in Europe.[1]

2. *A cold period in April.*—This is the "blackthorn winter" of England, so called because it is supposed to set in when the blackthorn is in blossom.[2] In this connection it may be noted that in various parts of the United States so-called "winters" are popularly associated with the flowering of the redbud, dogwood, snowball, and other plants. R. Abercromby,[3] who found evidence of a recurrent cold period in Scotland, April 11–14, shows that, taking account of the change from the Julian to the Gregorian calendar, this period coincides with the "borrowing days," the last three days of March, reputed in British folklore to have been borrowed by March from April, and notorious for cold and stormy weather.

3. *A cold period in May.*—In European weather lore this is the most celebrated of the periods under discussion. Over a considerable part of continental Europe it has been popularly believed since the Middle Ages that destructive frosts were likely to occur at a certain period in the month of May, and with the elaboration of the ecclesiastical calendar these frosts became definitely associated with the days dedicated to Saints Mamertus, Pancras, and Servatius (May 11, 12, 13), or, in south-central Europe, Saints Pancras, Servatius, and Boniface (May 12, 13, 14), hence known as the "ice saints." These saints and their days are called in French *saints de glace;* in German, *Eisheiligen*, *Eismänner*, or *gestrenge Herren*. In Bohemia, Pancras, Servatius, and Boniface are known collectively as Pan Serboni.

Passing mention should be made of the fact that in France the full moon which occurs late in April or early in May has a bad reputation as a bringer of frosts. It is known as the *lune rousse* ("russet moon"), in allusion to the brown appearance of frosted vegetation. Both the ice saints and the *lune rousse* evidently owe their notoriety to the fact that the beginning or early part of May is a critical period in the growth of vegetation, and frosts occurring at this time attract more attention than those which occur at other seasons.[4]

4. *A cold period in June.*—This depression in temperature is generally much more pronounced in European meteorological records than the cold period in May, but has not attracted public notice to the same extent because it is generally harmless to vegetation. It is recognized in German weather lore as the *Schafkälte* ("sheep-cold"); i. e., a chilly period dangerous to newly shorn sheep.[5]

5. The *dog days* (a period of heat after midsummer) doubtfully belong in this list. Various notions prevail as to their duration and time of occurrence, but in general they may be regarded as coinciding with the crest of the annual temperature curve, rather than with any irregularity therein.[6]

6. *Squaw winter.*—In the northern United States and Canada a period of wintry weather is reputed to precede Indian summer, and is known as "squaw winter."[7]

7. *A mild period in autumn*, especially in October and November; Indian summer of North America; St. Martin's summer, after-summer, old wives' summer, etc., of Europe. Typical Indian summer weather is calm, dry, and hazy or smoky, as well as warm for the season. The corresponding period in the Old World is associated in

[1] Messrs. W. M. Esten and C. J. Mason, in a discussion of a 21-year temperature record made at Storrs, Conn., find a sharp and prominent rise in the curves of both mean and extreme temperatures between the 20th and the 25th of January, which they identify with the "January thaw" of popular weather lore. (Storrs Agr. Exper. Sta. Bull. 64, September, 1910, p. 179.) There is, however, a surprising paucity of literature on this subject.

[2] J. Wright, "English dialect dictionary," 1, p. 284.

[3] Jour. Scott. meteor. soc., 2, 1867, p. 284.

[4] Moonlight at this period is correctly associated in the popular mind with frost, because a moonlit night is also a clear night, and hence favorable for nocturnal radiation. (See A. Angot, "Traité de météorologie," 3d ed., p. 398.) There is an abundance of other literature on the subject of the *lune rousse*.

[5] K. Almstedt, "Die Kälterückfälle in Mai und Juni," 1913, p. 5.

[6] As to the dates of the dog days, see the Oxford "New English Dictionary," s. v. *dog days*.

[7] The same name is occasionally applied to a spell of unseasonably cold or snowy weather in spring, *vide* Jour. Amer. Folk-lore, 20, 1907, p. 235.